CHAPTER 8

Coordinate Systems and Map Projections

E. Lynn Usery, Michael P. Finn and *Clifford J. Mugnier*

8.1 Introduction

Transformation of geographic data is necessary to support the development of a common coordinate framework from which geographic information system (GIS) operations, such as overlay, spatial buffering, and other analyses, can be performed. According to Keates (1982), we recognize three different types of transformations, the first two of which are mathematical and, therefore, reversible, and the third is non-mathematical and irreversible. The first of these transformations is from the spherical or ellipsoidal Earth to a plane coordinate system and is referred to as map projection. The second is transformation from the three-dimensional Earth form to a two-dimensional form. The final transformation is generalization from the real world to a representation and includes selection, simplification, symbolization and induction (Robinson et al. 1995). In this chapter, we will focus on the first and second types of transformations and the mathematical procedures that allow coordinate transformation to provide common reference frameworks for GIS. We also will briefly examine geometric correction of map and image data, which uses transformation of data from one plane coordinate system to another, but also is essential for GIS.

8.2 Geodesy

Transformation of the spherical or ellipsoidal surface of the Earth to a two-dimensional form falls under the fields of geodesy and map projections; areas of study with well-developed theory and implementation. In this chapter, only some basic aspects of this theory will be discussed to achieve the necessary basis for providing common frameworks for GIS. The study of spherical or ellipsoidal transformations from the Earth's surface to a two-dimensional representation requires the use of four interrelated concepts: ellipsoid, datum, map projection, and coordinate system. Each of these is discussed below.

8.2.1 Ellipsoids

The coordinate frame of reference for a geographic dataset is defined by a reference ellipsoid, a representation of the Earth in which the semi-major and semi-minor axes are of defined length (Figure 8-1). The term spheroid often is used synonymously with ellipsoid (Snyder 1987; Iliffe 2000); however, geodesists often use the terms separately reserving spheroid for association with a global datum on the ellipsoid. In this discussion we will use the term ellipsoid since this is the more common term for the basic figure of the Earth that is used for map projection. Common ellipsoids and their characteristics are shown in Table 8-1; a description of the correct use of the ellipsoids listed in Table 8-1 is given in Snyder (1987). A more complete listing of world ellipsoids is found in Mugnier (2004).

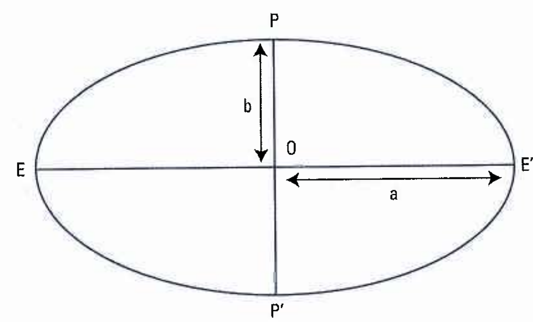

Figure 8-1 Terminology for ellipsoids of revolution: EE′ is the major axis; PP′ is the minor axis, a is the semi-major axis and b is the semi-minor axis.

Table 8-1 Selected official ellipsoids and their characteristics. Adapted from Snyder 1987.

Name	Date	Equatorial Radius (a) in meters	Polar Radius (b) in meters	Flattening (f)	Use
GRS 80	1980	6,378,137	6,356,752.3	1/298.257	Basis of NAD 83
WGS 72	1972	6,378,135	6,356,750.5	1/298.26	NASA; Dept. of Defense; oil companies
WGS 84	1984	6,378,137	6,356,752.3	1/298.257	Basis of GPS
Australian	1965	6,378,160	6,356,774.7	1/298.25	Australia
Krasovsky	1940	6,378,245	6,356,863.0	1/298.3	Soviet Union
Internationa l	1924	6 378,388	6,356 911.9	1/297	Remainder of the world
Hayford	1909	6 378,388	6,356 911.9	1/297	Remainder of the world
Clarke	1880	6,378,249.1	6,356,514.9	1/293.46	Most of Africa; France
Clarke	1866	6,378,206.4	6,356,583.8	1/294.98	North America; Philippines
Airy	1830	6,377,563.4	6,356,256.9	1/299.32	Great Britain
Bessel	1841	6,377,397.2	6,356,079.0	1/299.15	Central Europe; Chile; Indonesia
Everest	1830	6,377,276.3	6,356,075.4	1/300.80	India; Burma; Pakistan; Afghan; Thailand; etc.

Since the Earth is properly represented as an oblate ellipsoid (spheroid), the primary parameters defining the geometric representation are the Equatorial radius (a) and the Polar radius (b) (Table 8-1, Figure 8-1). Using a and b, we define the flattening factor (f) as:

$$f = (a-b)/a \tag{8.1}$$

The flattening factor is a measure of the oblateness of the ellipsoid and since an approximate factor for the Earth is 1/298, it is not visible to the naked eye even in satellite views. Therefore, the oblateness shown in Figure 8-1 is exaggerated to allow the visual interpretation of the ellipsoid shape. We define the first eccentricity (e), another fundamental measure of the shape (characteristic) of an ellipsoid of revolution, from the flattening factor as:

$$e = (2f - f^2)^{1/2} \tag{8.2}$$

We can then define the geodetic coordinates — latitude (φ), longitude (λ), and height above the ellipsoid (h) — as shown in Figure 8-2. Geodetic coordinates may be transformed to Earth-centered Cartesian coordinates, X, Y, and Z using the following equations:

$$X = (\nu + h) \cos \varphi \cos \lambda \tag{8.3}$$

$$Y = (\nu + h) \cos \varphi \sin \lambda \tag{8.4}$$

$$Z = \{1-e^2)\, \nu + h\} \sin \varphi \tag{8.5}$$

where

$$\nu = a/(1-e^2 \sin^2 \varphi)^{1/2} \tag{8.6}$$

For complete mathematical development of geodetic coordinates and transformations with Cartesian coordinates, see Mugnier (2004).

The most common ellipsoids currently (2007) used with geographic data are Clarke 1866, World Geodetic System 1984 (WGS 84), and Geodetic Reference System 1980 (GRS 80). The Clarke 1866 ellipsoid is the basis for most maps created in the US before the

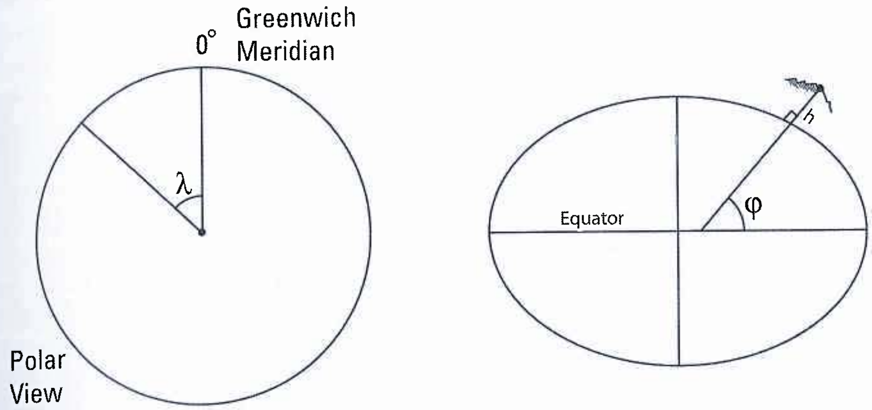

Figure 8-2 Representation of geodetic coordinates, latitude (φ), longitude (λ), and height above the ellipsoid (h).

1980s, primarily because it was designed to fit North America. The Clarke 1866 ellipsoid is referenced to the Earth's surface with geodetic measurements and is the basis of the North American Datum of 1927 (NAD 27). The WGS 84 and GRS 80 ellipsoids were established by satellite positioning techniques, are referenced to the center of mass of the Earth, i.e., geocentric, and provide a reasonable fit to the entire Earth. The WGS 84 datum provides the basis of coordinates collected from the Global Positioning System (GPS), although modern receivers transform the coordinates into almost any user selected reference datum.

8.2.2 Datums

A datum is the basis of a coordinate system and defines an initial point. A datum can be local or global depending on the initial point and whether or not the datum is referenced to an ellipsoidal representation of the Earth. A horizontal datum allows specification of latitude and longitude or x, y Cartesian coordinate locations relative to the initial point. A vertical datum allows specification of height above or below the initial point. For a global datum, the initial point is a point on the surface of the Earth, as with NAD 27, which uses the triangulation station at Meads Ranch, Kansas, as an initial point. Such a datum is referenced as a geodetic datum, and requires another point to establish a reference angle to align the coordinate system. For NAD 27, the reference point is the nearby triangulation station, Waldo in Kansas. For a geocentric datum, established by satellite positioning, the initial point is the center of the Earth and no reference angle is required (Snyder 1987; Iliffe 2000). For detailed treatment of datum concepts, including complete mathematical development, see Mugnier (2004).

8.3 Coordinate Systems

We can define an ellipsoidal coordinate system called the geographical reference system (Figure 8-3) of latitude (φ) and longitude (λ) once we have defined the datum. Note that each ellipsoidal system is different based on the choice of datum; thus, a specification of latitude and longitude location is not sufficient without knowing the datum. Differences in projected plane coordinates can be hundreds of meters as in the United States where the difference between Universal Transverse Mercator (UTM) coordinates on NAD 27 and NAD 83 may be as much as 200 m (Welch and Homsey 1997).

With a datum and projection defined, we can then define plane coordinate systems. A coordinate system uses the initial point of the datum in the projection chosen, and establishes X and Y coordinates based on a grid system of the selected units of measure. Common plane coordinate systems are based on a set of Cartesian axes usually referenced as X and Y

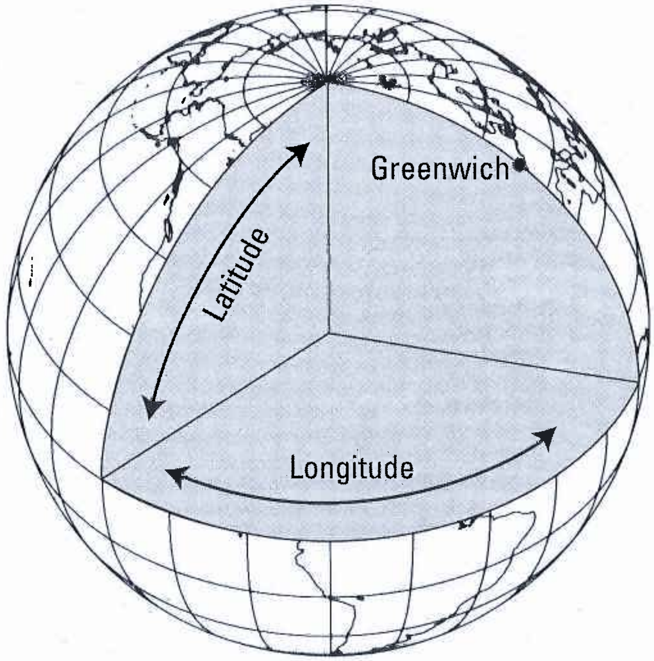

Figure 8-3 Geographic reference system with coordinates of latitude and longitude.

or Eastings (E) and Northings (N) with units measured in feet or meters. A right-handed Cartesian coordinate system defines the origin at 0, 0 and X increases to the right and Y increases to the top. Two common US systems for large-scale (high-resolution) applications are the UTM, a worldwide system, and the State Plane Coordinate systems of the United States, its territories and possessions.

In map projection terminology, we define the scale factor at the origin, m_o, as the maximum scale distortion in the projection. It is the ratio of the scale along a meridian or parallel at a given point to the scale at a standard point or along a standard line that is made true to scale (Snyder 1987). The UTM system is based on projections of six-degree zones of longitude, 80° S to 84° N latitude and the scale factor is specified for the central meridian of the zone.[1] The scale factor for each UTM zone along the central meridian of the projection is 0.9996, yielding a maximum error of 1 part in 2,500. In the northern hemisphere, the X coordinate of the central meridian is offset to have a value of 500,000 meters instead of 0, normally termed the "False Easting." The Y coordinate has 0 set at the Equator. In the southern hemisphere the False Easting is also 500,000 meters with a Y offset of the Equator or False Northing equal to 10,000,000 meters. These offsets force all coordinates in the system to be positive.

The State Plane Coordinate system, available only in the United States and its territories and possessions, also uses Eastings and Northings as the coordinate axes. It also is projected in zones to preserve accuracy with State Plane Coordinate zones designated by states. The maximum width of a zone (the part of the Earth surface projected with its own unique central meridian) is 158 miles wide, which allows a higher accuracy of transformation from the ellipsoid to the plane than the UTM system. The zone width allows maintenance of a m_o

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1. In the Universal Military Grid System, the polar areas, north of 84° N and south of 80° S, are projected to the Universal Polar Stereographic Grid with the pole as the center of projection and a m_o = 0.9994. They are termed "North Zone" and "South Zone."

of approximately 0.9999, or an accuracy of 1 in 10,000. The projection from the ellipsoid also is dependent on the shape of the state. States with an east-west long axis, Tennessee, for example, use the Lambert Conformal Conic projection for each zone. States with a north-south long axis, Illinois, for example, use the Transverse Mercator projection for each zone. Three states, New York, Florida, and Alaska, use both projections since these states have parts extending both E–W and N–S. Zone 1 of Alaska uses the Hotine Rectified Skew Orthomorphic (RSO) Oblique Mercator projection. Coordinate measurement units of State Plane Coordinates depend on the datum. For NAD 27, the measurement units are US Survey Feet (as opposed to the International Foot defined as 610 nm smaller than the US Survey Foot); while newer systems cast on NAD 83 have an official unit of the meter. Often NAD 83 coordinates also are expressed in feet, but depending on the state, some now use US Survey Feet, and others the International Foot. Some states, such as Wisconsin and Minnesota, have established plane coordinate systems for each county specifically for use with GIS applications. The traditional m_o at the origin, normally associated with the State Plane Coordinate system zones, is modified for height above the ellipsoid so that field survey measurements will correspond closely with the county GIS scale factor and thus reduce hand computations necessary for conversion by the GIS analyst.

A final plane coordinate system of relevance to geographic data synthesis and modeling, particularly for satellite images and photographs, is an image coordinate system. A digital image system is not a right-handed Cartesian coordinate system since usually the initial point (0,0) is assigned to the upper left corner of an image. The X coordinate, often called sample, increases to the right, but the Y coordinate, called the line, increases down. Units commonly are expressed in picture elements or pixels. A pixel is a discrete unit of the Earth's surface, usually square with a defined size, often expressed in meters. Photogrammetric applications, however, transform the origin (0,0) of each image to correspond to the center of perspective, or the intersection of the optical axis of the lens with the image plane. For frame imagery, that is the center of the image. For pushbroom imaging sensors, different geometry is used and modeled, and commonly is associated with rational functions.

8.4 Map Projections

Since the Earth is spherical or, more correctly, ellipsoidal, and usually we work with plane coordinate representations, geographic data must be projected from ellipsoidal coordinates to plane coordinates. This transformation is referred to as map projection, which is defined as a systematic transformation of ellipsoidal coordinates of latitude and longitude to a plane coordinate representation and mathematically,

$$x = f_1(\varphi,\lambda) \tag{8-7}$$

$$y = f_2(\varphi,\lambda) \tag{8-8}$$

The transformation is implemented from a "generating globe," which is a reduced scale model of the Earth as either a sphere, an ellipsoid, or an "aposphere." The projection transformation always results in error, with only a single point, circle, or one or two lines where the scale relation to the generating globe is true. While error always is a result of the transformation, specific properties can be preserved, e.g., angular relations in small areas, polygon areas, such as continents, or specific directions, such as straight lines away from the North or South Poles. Angles and area, however, cannot be preserved simultaneously in the same projection since they are mutually exclusive transformations. Maps with angles preserved are called conformal projections. Maps with areas preserved are called equal-area or equivalent projections. Equal area projections also are called authalic, meaning that at any point the scales in two orthogonal directions are inversely proportional, which forces equal areas.

One can understand map projection by examining the transformation of spherical coordinates to the geometric figures: cylinder, cone, and plane. A graphical illustration of these transformations is shown in Figure 8-4. Note that for a cylinder only a single line of contact exists in a tangent projection (two lines for a secant projection). It is only along this line that the true scale of the generating globe, and thus along the same imaginary line on the Earth's surface, is retained. This line is said to have a scale factor of 1. Any other line is projected away from the sphere and possesses a scale factor larger than 1. Note that the greater the distance away from the line of contact, the Equator in a normal aspect cylindrical projection, the larger the scale factor and thus the greater the distortion. A similar line for a conic projection is shown in Figure 8-4, but for an azimuthal (zenithal) projection, only a single point on the tangent plane retains a scale factor of 1. Complete documentation of map projection theory is available in Snyder (1987), Pearson II (1990), Bugayevskiy and Snyder (1995), Yang et al. (2000) and Canters (2002). A history of map projection development is available in Snyder (1993) and a documentation of characteristics of various projections is provided in Snyder and Voxland (1989).

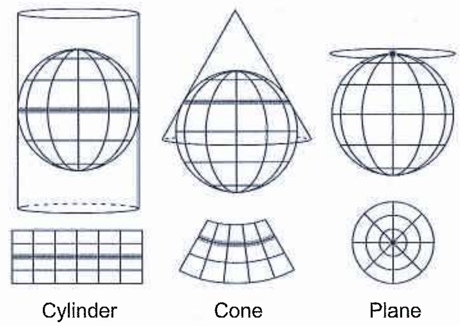

Figure 8-4 Geometric figures used for map projection from sphere or ellipsoid to the plane. All are shown in the normal aspect with lines or points where the scale factor = 1 is highlighted.

8.4.1 Classification

Projections are classified by a variety of methods including geometry, shape, special properties, projection parameters and nomenclature (Canters 2002). The geometric classification is based on the patterns of the graticule, the grid of parallels of latitude and meridians of longitude, that result from a perspective projection of the sphere on a cylinder, cone or plane as shown in Figure 8-4. These projections are referred to as cylindrical, conical, azimuthal (also occasionally called zenithal) and aphylactic (meaning none of the former). Although commonly referred to as developable because of the apparent ability to develop these projections from a perspective projection of the sphere, the spacing of the parallels in the patterns is derived from differential calculus. This process allows for the preservation of specific characteristics and minimizes distortion, such as angular relations (shape) or area. The geometric classification into cylindrical, conical and azimuthal is not complete since many projections fit none of these classes. Thus, the classification commonly is expanded to include pseudocylindrical, projections with straight parallels as with the cylindricals, but curved meridians; pseudoconical with parallels as curved arcs, as with the conicals, but with parallel length adjusted so meridians are not straight arcs; and polyconical, with non-concentric circular arcs for the parallels (Canters 2002). Projections that do not fit these six classes are referred to as non-conicals. A complete description of these geometric patterns and their associated names can be found in Lee (1944).

Another common projection classification system is based on the shape of the graticule. Maurer (1935), cited in Canters (2002), developed a hierarchical system including five levels primarily based on the appearance of the meridians and parallels. For a description of the system, see Maurer (1935) or Canters (2002). Starostin et al. (1981) also presented a classification system based on the shape of the graticule as described in Bugayeskiy and Snyder (1995). This system is similar to Maurer's with classes based on the shape of the parallels and symmetry of the graticule.

Subdivisions in Lee's (1944) classification are based on special properties. Goussinsky (1951) also produced a system based on special projection properties. Within his system, the five classes—nature, coincidence, position, properties and generation—are not mutually exclusive, but within each class, the types of properties preserved are exclusive. For example, class 3, position, includes direct, transverse and oblique. Maling (1968) proposed 11 special properties to use in map projection.

Tobler (1962) used a general approach based on parametric classification in four groups. The groups are based on whether or not the plane coordinates x and y are based on a time, formation of latitude, longitude, or both. Maling (1992) included geometric classes that relate to Tobler's parametric classes and the traditional geometric approach. A complete description of Tobler, Maling and other classification systems is available in Canters (2002). As is obvious from this brief discussion, map projections can be classified in many ways. The International Cartographic Association (ICA) Commission on Map Projections has a current (2006) project to establish a standard classification and naming system (ICA 2006).

8.4.2 Suggested Projections

The selection of an appropriate map projection for a given application depends on a variety of factors, including the purpose the map, the type of data to be projected, the area of the world to be projected and scale of the final map. Advice on selection is available from a variety of print and web sources, including Finn et al. (2004) and USGS (2006). In the discussion below that provides a description of specific projections, we will distinguish between large-scale (small areal extent) and small-scale (large areal extent) applications. In GIS, large-scale data sets commonly are projected with a conformal projection to preserve angles and shape. For such applications, area distortion is so small over the geographic extent that it is negligible and an area preserving projection is not needed. Whereas there is no sharp boundary to determine large-scale from small-scale applications, we will use an area of 150,000 square kilometers, roughly the size of some US states, and a scale of 1:500,000 as a convenient breakpoint. Commonly, large-scale data files are used in GIS applications of limited geographic extent, e.g., a watershed, a county or a state. The two most commonly used projections for these scales are the Lambert Conformal Conic and the Transverse Mercator, which are the basis of the UTM and most of the State Plane coordinate systems discussed earlier in this chapter. Later in this chapter, we will describe these projections and several others used for small-scale applications for areal extents of states, regions, countries, continents or the entire globe. The descriptions are adapted and dates of presentation and authors are taken from Snyder and Voxland (1989) unless otherwise noted.

8.4.3 Description of Specific Projections

The following section details characteristics of a selected set of projections. The name and creator, or inventor, is detailed for each projection. We outline specific characteristics including properties preserved, shapes of parallels and meridians, the lines or points of true scale, the extent of the Earth that can be shown and—for specific cases—particular characteristics that make the projection unique. A graphic representation of each projection is included with world data and a set of distortion circles plotted on the graphic, usually for one quarter of the Earth coverage, since the distortion circles repeat the same patterns in other quadrants. The distortion is plotted as a Tissot Indicatrix (Tissot 1881; Canters 2002). The intersection of any two lines on the Earth is represented on a map with an intersection at the same or different angles (Figure 8-5). At almost every point, there is a right angle intersection of two lines in some direction, which also is shown as a right angle on the map. All other line intersections at that point will not be at right angles, unless the map is conformal at that point. The greatest deviation from the correct angle is the maximum angular deformation (ω). For a conformal map, the value of ω is zero. We use plots of small circles on the maps to indicate distortion; if the circles all remain circles, but change in size, the map is conformal and does not preserve area. If the circles change shapes to ellipses of the same size, the map preserves area, but does not maintain angular relations. On maps with changes in both the shape and size of the circles, neither area nor angular relations are preserved. The reader should note that in the projection figures the circles and ellipses may not appear to be the same area when in fact they are. It is well-known that the human eye is poor at estimating the relative size of geometric symbols, such as circles and ellipses, and methods to psychologically scale such symbols have been developed and used in cartography (Flannery 1971). However, in the figures in this chapter, exact areas are used without psychological scaling, and thus some circles and ellipses will appear to be of different size when in fact they are equal.

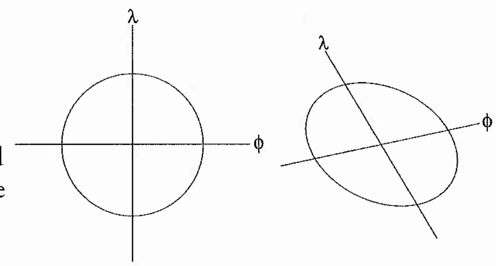

Figure 8-5 A graphic illustration of Tissot's Indicatrix, right. An infinitely small circle on the Earth (left) appears as an ellipse on many maps. Only on conformal maps will the figure remain a circle on the map.

8.4.3.1 Cylindrical

8.4.3.1.1 Mercator

The Mercator projection is a cylindrical conformal projection developed by Gerardus Mercator in 1569. It was developed to show loxodromes or rhumb lines, which are lines of constant bearing, as straight lines. The Mercator projection made it possible to navigate a constant course based on drawing a rhumb line on the chart. The projection has meridians as equally spaced parallel lines, while parallels are shown as unequally spaced straight parallel lines, closest near the Equator and perpendicular to the meridians. The North and South Poles cannot be shown. Scale is true along the Equator (tangent case) or along two parallels equidistant from the Equator (secant case). Significant size distortion occurs in the higher latitudes as shown by the circle sizes in Figure 8-6. The Mercator projection was defined for navigational charts and is best used for navigation purposes. It is a standard for marine charts.

8.4.3.1.2 Transverse Mercator

The transverse aspect of the Mercator projection is a projection where the line of constant scale is along a meridian rather than the Equator. The central meridian, each meridian 90° from the central meridian, and the Equator are straight lines. Other meridians and parallels are complex curves, concave toward the central meridian and nearest pole, respectively. The Poles are points along the central meridian. The projection has true scale along the central meridian or along two meridians equidistant from and parallel to the central meridian in the secant case. Conceptually, it is created by projecting onto a cylinder wrapped around the globe tangent to the central meridian or secant along two small circles equidistant from the central meridian. It commonly is used for large-scale, small area, presentations; many of the world's topographic maps from 1:24,000 scale to 1:250,000 scale use this projection. It is the basis of the UTM coordinate system and many of the State Plane Coordinate systems for states with an elongated north-south axis. The Transverse Mercator projection using the zero degree longitude at Greenwich as the central meridian is shown in Figure 8-7.

8.4.3.1.3 Lambert Cylindrical Equal Area

The Cylindrical Equal Area projection, first presented by Johann Heinrich Lambert in 1772, became the basis for many other similar equal area projections including the Gall Orthographic, Behrmann, and Trystan-Edwards. From Lambert's original projection with the line

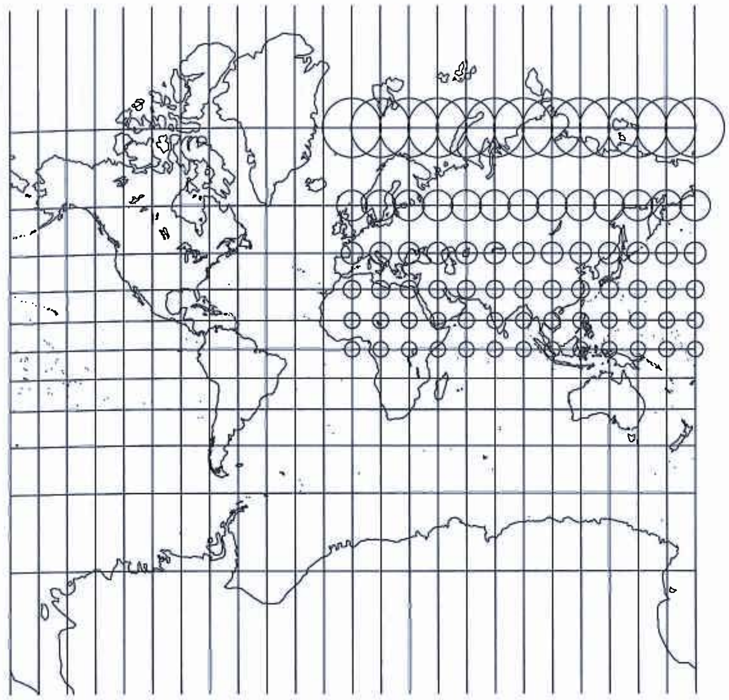

Figure 8-6 Mercator projection with distortion circles (Tissot's Indicatrix) in the upper right corner, illustrating that size distortion is greater at higher latitudes.

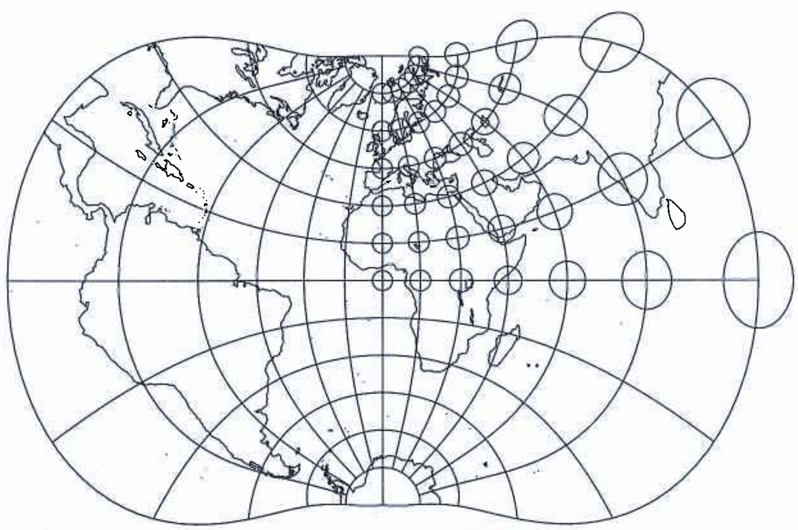

Figure 8-7 Transverse Mercator projection with the central meridian through Greenwich; Tissot's Indicatrices in the upper right illustrate that scale is constant along the prime meridian, and how distortion in this projection is greater for locations farther from the Equator and central meridian.

of constant scale along the Equator, one simply makes the projection secant at two small circles (parallels). Each of the above projections uses different parallels as the lines of constant scale. Lambert's Cylindrical Equal Area projection has meridians that are equally-spaced straight parallel lines 0.32 times as long as the Equator. Lines of latitude are unequally spaced parallel lines furthest apart near the Equator, and are perpendicular to the meridians. The projection maintains equal areas by changing the spacing of the parallels. Significant shape distortion, however, results from maintaining equal areas with the distortion greater in high latitudes near the poles as shown by the ellipses in Figure 8-8. While this projection is not often used, it is a standard to describe map projection principles in textbooks. It has also served as a prototype for other projections, as described earlier in this chapter.

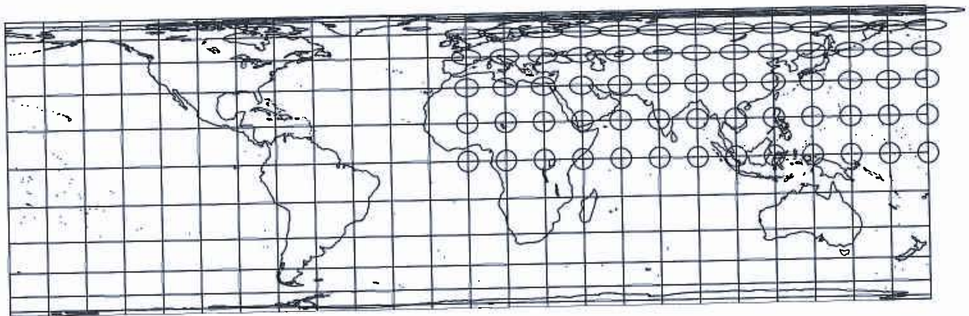

Figure 8-8 Lambert Cylindrical Equal Area projection.

8.4.3.2 Conical

8.4.3.2.1 Lambert Conformal Conic

The Lambert Conformal Conic (LCC) projection, presented in 1772, shows meridians as equally spaced straight lines converging at a common point, which is one of the poles. Angles between the meridians on the projection are smaller than the corresponding angles on the globe. Parallels are unequally spaced concentric circular arcs centered on the pole of convergence of the meridians, and spacing of the parallels increases away from the pole. The pole nearest the standard parallel is a point; the other pole cannot be shown. Scale is true along the standard parallel or along two standard parallels in the secant case. Scale also is constant, although not true, along any given parallel. The projection is free of distortion only along one or two standard parallels. Shapes are maintained at the expense of area as shown by the perfect circles of different sizes in Figure 8-9. The LCC projection is extensively used for large-scale mapping of regions with an elongated axis in the east-west directions and in mid-latitude regions. It is the projection for the State Plane Coordinate system for US states with an east-west axis, such as Tennessee. It also is a standard of the US Geological Survey (USGS) for State Base Maps at 1:500,000 scale, and for maps of the 48 US contiguous states.

8.4.3.2.2 Albers Equal Area

The Albers Equal Area projection, presented by Heinrich Christian Albers in 1805, has meridians as equally spaced straight lines converging at a common point, which normally is beyond the pole. Angles between the meridians are less than the true angles and parallels are unequally spaced concentric circular arcs centered on the point of convergence of the meridians. Spacing between the parallels decreases away from the point of convergence, the poles being circular arcs. Scale is true along one or two standard parallels. The scale factor at any given point along a meridian is the reciprocal of the scale factor along the parallel, thus preserving area at the expense of shape. This is shown in Figure 8-10 where circles maintain size but change in shape to ellipses away from the standard parallel. The projection is free of angular and scale distortion only along the one (tangent case) or two (secant case) standard parallels. The Albers Equal Area projection is used to show areas of east-west extent in applications where preservation of area is important. It commonly is used for equal-area maps for the 48 contiguous US states and is the projection upon which the *National Atlas of the United States* (www.nationalatlas.gov) is based.

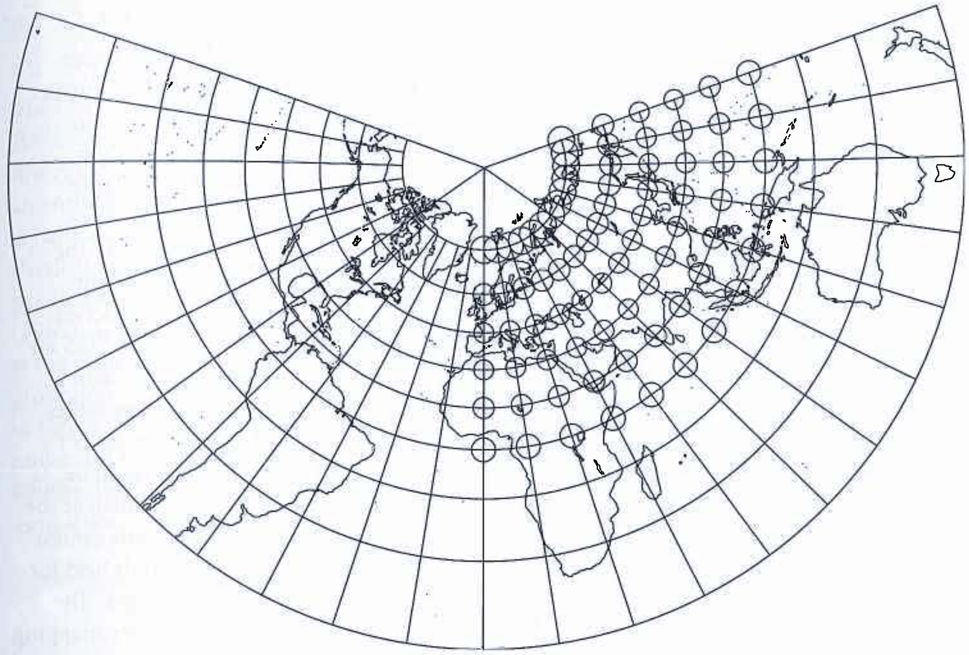

Figure 8-9 Lambert Conformal Conic projection preserves shape, as shown by the fact that Tissot's Indicatrix is everywhere a circle. Variation in the size of circles shows that area is not preserved.

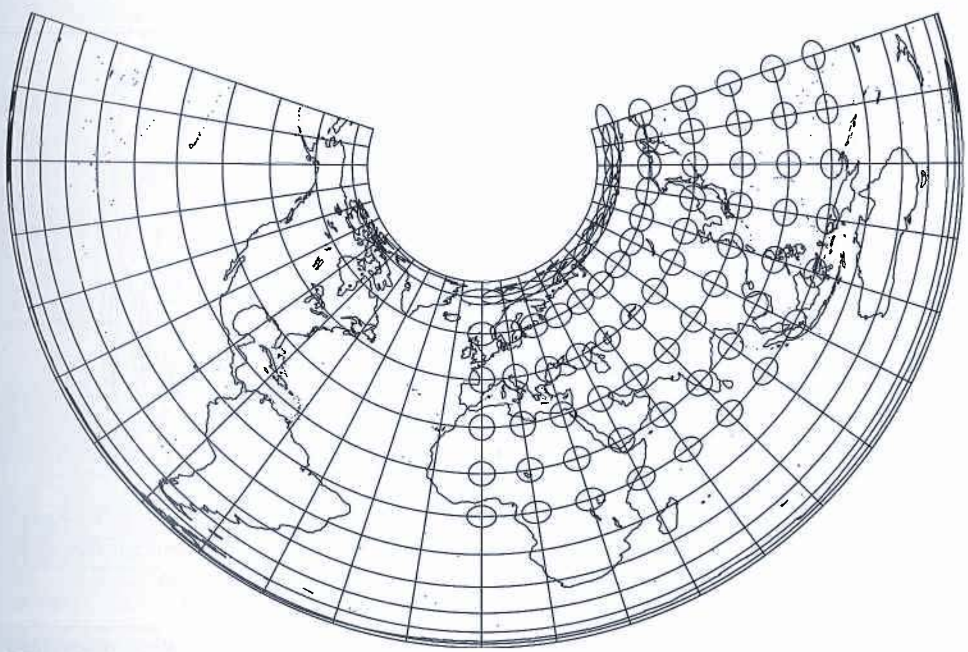

Figure 8-10 Albers Conical Equal Area projection.

8.4.3.2.3 Polyconic

The polyconic projection was originated by Ferdinand Rudolph Hassler of the US Coast and Geodetic Survey for plane table and alidade coastal mapping, and was easy to construct from simple tables while in the field. The projection uses many cones for the projection, one along each parallel, hence the name "poly" conic. The central meridian is a straight line with all others appearing as complex curves. The Equator is the only parallel that is a straight line, with others as non-concentric circular arcs spaced at true distances along the central meridian. Scale is true along the central meridian and along each parallel. The projection is free of distortion only along the central meridian and results in significant distortion if the range is extended far to the east and west. While the projection preserves neither area nor shape (termed aphylactic), it was the only projection used by the USGS for topographic maps until the 1950s. One reason for this usage was the ease of construction of the projection for quadrangle maps from tables of rectangular coordinates. These tables may be used from any polyconic projection on the same ellipsoid by applying the proper scale and central meridian. Therefore, for each quadrangle map the same tables could be used. These quadrangle maps for the same ellipsoid and for the same central meridian at the same scale will fit exactly from north to south. They also fit exactly east to west, but cannot be mosaicked in both directions simultaneously unless only one central meridian is held for an entire map series. Such variations of aphylactic projections are called quadrillages. The Polyconic projection also was used for the Progressive Military Grid for the military mapping of the United States in the 15-minute format. This grid later was incorporated into the World Polyconic Grid that was referenced to the Clarke 1866 ellipsoid, measured in yards, and used for artillery fire control mapping during World War II. A graphic illustration of the projection is shown in Figure 8-11. This projection is not recommended for regional or global maps since better projections are available.

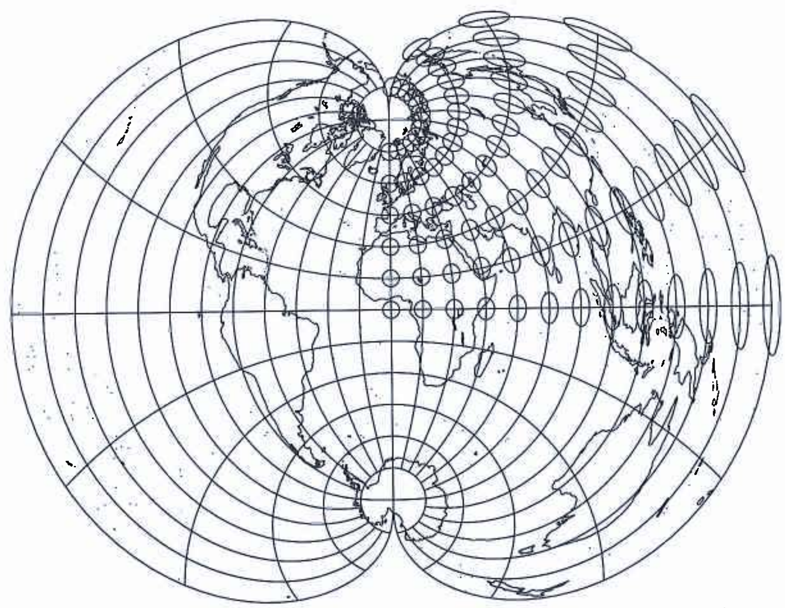

Figure 8-11 The Polyconic projection applied to a world data set.

8.4.3.3 Azimuthal

8.4.3.3.1 Orthographic

The Orthographic projection, developed by the Egyptians and Greeks by the 2nd century B.C., is a perspective azimuthal (planar or zenithal) projection that is neither conformal nor equal-area. It is used in polar, Equatorial and oblique aspects, and results in a view of an entire hemisphere of the Earth. In the polar aspect, shown in Figure 8-12, meridians are equally spaced straight lines intersecting the central pole. Angles between meridians are true. Parallels are unequally spaced circles centered on the pole, which is a point. Spacing of the parallels decreases away from the pole. Other aspects are described in Snyder and Voxland (1989). Scale is true at the center and along the circumference of any circle with its center at the projection center. Such circles are parallels in the polar aspect of the orthographic projection. Scale decreases radially with distance from the center. Distortion circles are shown in Figure 8-12, which also shows the globe-like look of the projection. The orthographic projection is essentially a perspective projection of the globe onto a tangent plane from an infinite distance (orthogonally). It is commonly used for pictorial views of the Earth as if seen from space.

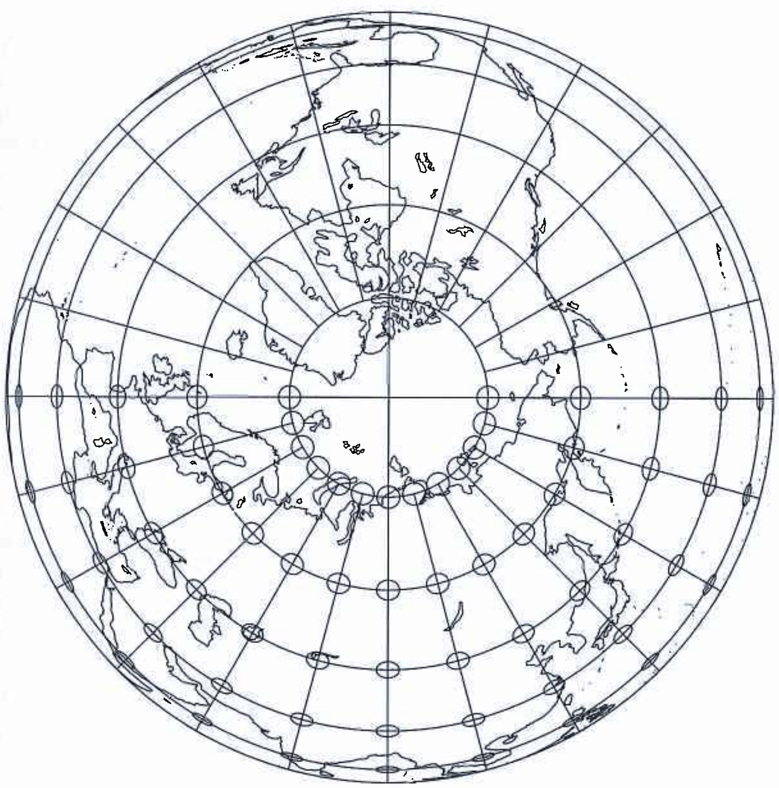

Figure 8-12 The Orthographic projection in the polar aspect.

8.4.3.3.2 Stereographic

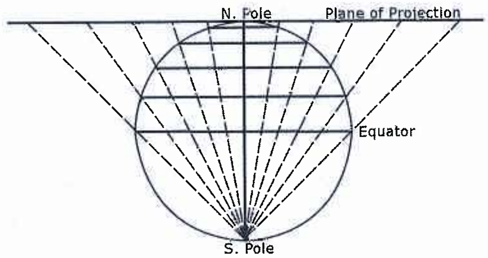

Figure 8-13 Projection from the South Pole onto a plane tangent at the North Pole creates the Stereographic projection.

The Stereographic projection, also developed by the Egyptians and Greeks by the 2nd century B.C., is a perspective azimuthal projection that preserves angles, i.e., is conformal. As with the Orthographic projection, the polar, Equatorial and oblique aspects result in different appearances of the graticule. The polar aspect is achieved by projecting from one pole to a plane tangent at the other pole (Figure 8-13). In this aspect, meridians are equally-spaced straight lines intersecting at the pole with true angles between them. Parallels are unequally spaced circles centered on the pole represented as a point. Spacing of the parallels increases away from the pole. The projection commonly is used only for a hemisphere. It can be used to show most of the other hemisphere (Figure 8-14) at an accelerating scale. Scale is true only where the central latitude crosses the central meridian or along a circle concentric about the projection center, and scale is constant along any circle with the same center as the projection. The Stereographic projection is used in the polar aspect for topographic maps of the polar regions. The Universal Polar Stereographic is the sister projection to the UTM for military mapping. This projection is in current (2007) use in oblique ellipsoidal form in a number of nations throughout the world, including Canada, Romania, Poland and The Netherlands (Thompson et al. 1977). This projection generally is chosen for regions that are roughly circular in shape, and it normally is used only in the secant case where the scale factor is less than 1.0. Different countries have different mathematical developments that include the Stereographic Double, the Roussilhe Stereographic, and various truncations of the Hristow Stereographic. East and West hemisphere maps commonly use the Equatorial aspect of the Stereographic projection.

8.4.3.3.3 Gnomonic

The Gnomonic projection is a perspective azimuthal projection that is neither conformal nor equal area. The Greek, Thales, possibly developed it around 580 B.C. The name derives from the point of projection being at the center of the earth where the mythical "gnomes" live. It has the unique feature that all great circles, including all meridians and the Equator, are shown as straight lines. As with other azimuthals, the graticule appearance changes with the aspect. In the polar aspect, meridians are equally spaced straight lines intersecting at the pole with true angles between them. Parallels are unequally spaced circles centered on the pole as a point. Spacing of the parallels increases from the pole. The Equator and opposite hemisphere cannot be shown. The projection, which can be viewed conceptually as projected from the center of the globe on a plane tangent at a pole or another point, only can show less than a hemisphere. Scale is true only where the central parallel crosses the central meridian and increases rapidly with distance from the center. Distortion circles (Figure 8-15) show that the projection is neither conformal nor equal area. Its usage results from the special feature of great circles as straight lines, and thus assists navigators and aviators in determining the shortest and most appropriate courses.

8.4.3.3.4 Lambert Azimuthal Equal Area

The Lambert Azimuthal Equal Area projection, developed by Johann Heinrich Lambert in 1772, is a non-perspective azimuthal equal area projection. In the polar aspect, meridians are equally spaced straight lines intersecting at the central pole with true angles between them. Parallels are unequally spaced circles centered at the pole as a point. Parallel spacing decreases away from the pole. The projection can be used for the entire Earth with the opposite pole appearing as a bounding circle with a radius 1.41 times that of the Equator. Scale is true at the center in all directions and decreases rapidly with distance from the center along radii and

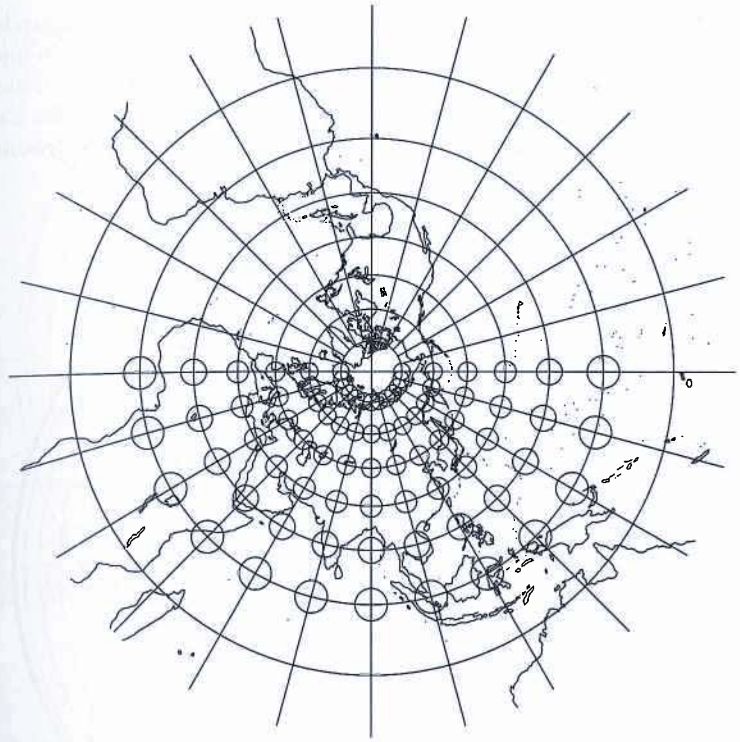

Figure 8-14 Stereographic projection showing more than one hemisphere of data.

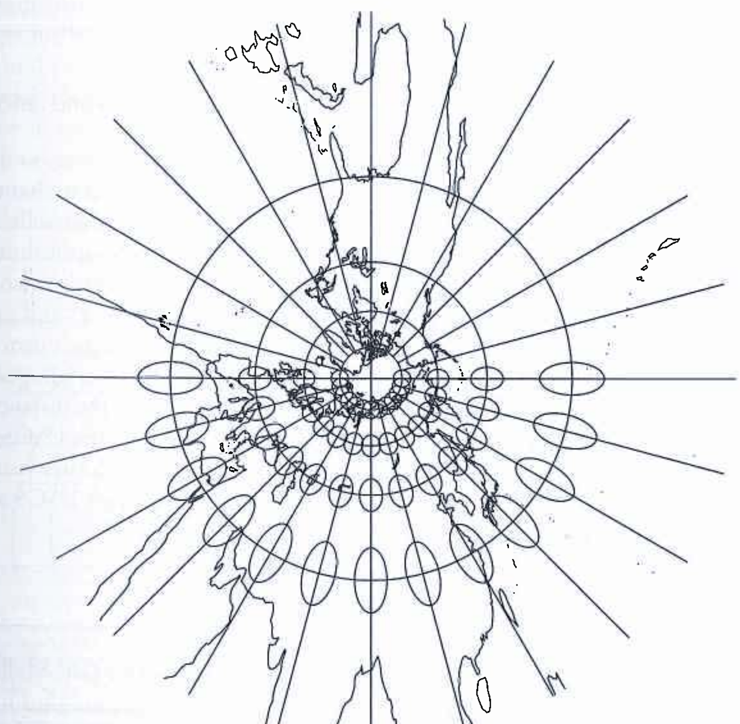

Figure 8-15 The Gnomonic projection in the polar aspect. The variation is both size and shape of the Tissot's Indicatrices show that it does not preserve either.

increases with distance in a direction perpendicular to radii. A projected northern hemisphere with distortion circles showing the equal area preservation, but shape distortion into ellipses is shown in Figure 8-16. The Lambert Azimuthal Equal Area projection often is used in the polar aspect for atlases of the polar regions. The Equatorial aspect is used for the East and West hemisphere maps, and is best used for equal-area maps of regions of approximately circular extent with the projection centered on the center point of the region.

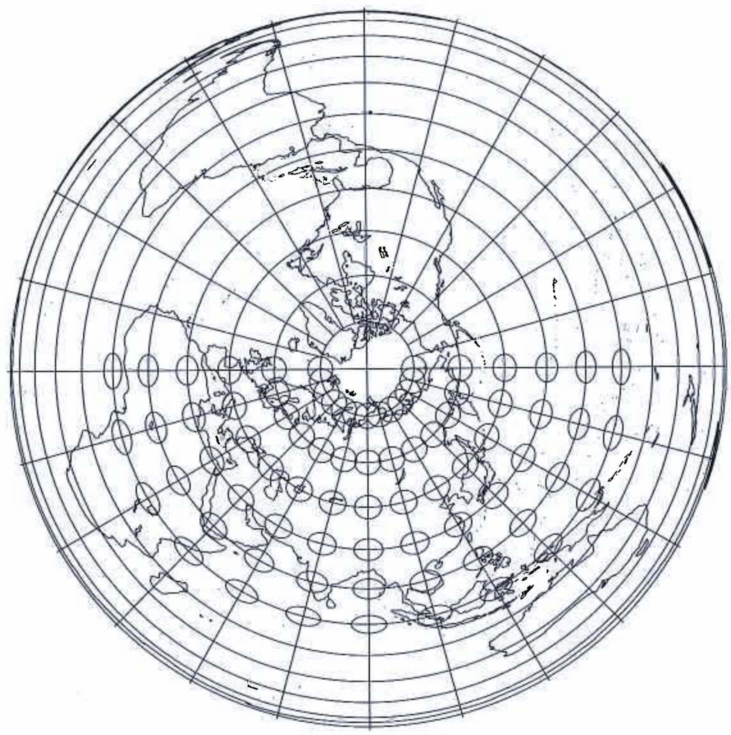

Figure 8-16 The Lambert Azimuthal Equal Area projection preserves area and distorts shape.

8.4.3.3.5 Azimuthal Equidistant

The (ellipsoidal) Hatt Azimuthal Equidistant projection has origins similar to that of the Roussilhe Oblique Stereographic. Both Philippe Eugene Hatt and Henri Roussilhe were Chief Hydrographers of the French Navy, and both men devised projections for use in the hydrographic surveys of near-shore waters and harbors. Because of the prestige associated with the papers published by both men in *Annals Hydrographique* in the 19th and early 20th centuries, a number of countries adopted one or the other projection for their own grids (Takos 1978). The Hatt Azimuthal is merely based on a polar coordinate origin point from which clockwise azimuths from north are measured to points. To define the distance to the points specific series expansions for the geodesic are used (Figure 8-17). The USGS used a modified Azimuthal Equidistant for geological mapping of Yemen, and in later years, John P. Snyder used Clarke's Long-Line Geodesic for the Azimuthal Equidistant USGS series of Micronesia.

8.4.3.4 Pseudocylindrical

8.4.3.4.1 Mollweide

The Mollweide is a pseudocylindrical equal area projection developed by Carl Mollweide in 1805. The central meridian is a straight line one-half as long as the Equator, thus forming an elliptical area of projection for the entire globe. Meridians 90° East and West of the central meridian form a circle. Other meridians are equally spaced semi ellipses intersecting at the

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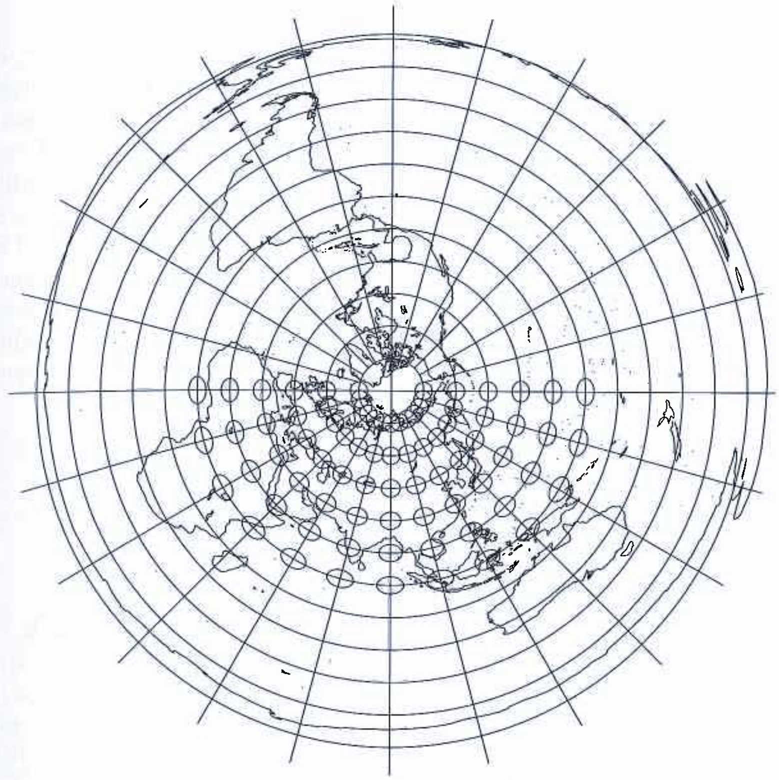


Figure 8-17 The Azimuthal Equidistant projection in a polar aspect.

poles and concave toward the central meridian. Parallels are unequally spaced straight parallel lines perpendicular to the central meridian, farthest apart near the Equator with spacing changing gradually. The poles are shown as points. Scale is true along latitudes 40°44' North and South and constant along any given latitude. The entire globe projected and centered on the Greenwich meridian is shown in Figure 8-18. The distortion circles indicate preservation of area since all are the same size, and distortion of shape since they become ellipses toward the poles. It occasionally has been used for world maps, particularly thematic maps where preservation of area is important. Goode (1925) combined it with the sinusoidal projection to create the Homolosine. Different aspects of the Mollweide have been used for educational purposes, and the projection was used in *The Times Atlas* in England.

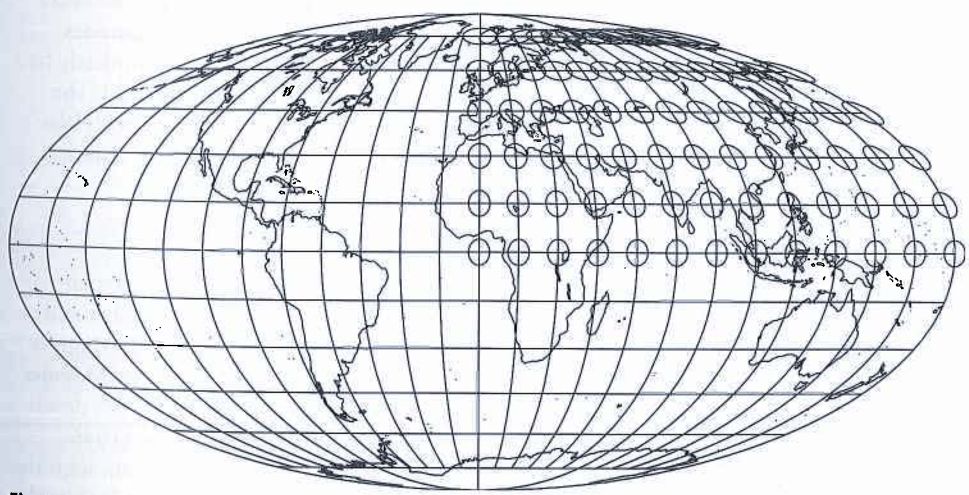


Figure 8-18 The Mollweide projection.

8.4.3.4.2 Sinusoidal

The Sinusoidal is an equal-area, pseudocylindrical projection developed in the 16^{th} century and used by various cartographers in atlases. It also is known as the Sanson-Flamsteed projection for later users and is the oldest of the pseudocylindrical projections. The central meridian is a straight line one-half as long as the Equator. Other meridians are equally spaced sinusoidal curves intersecting at the North and South Poles and concave toward the central meridian. The parallels are equally spaced straight lines perpendicular to the central meridian. The Poles are shown as points. The scale is true along the central meridian and along every parallel. The sinusoidal projection preserves area, but distorts shapes (Figure 8-19), with the greatest distortion occurring near outer meridians and in high latitudes. The Equator is free of distortion. It has been used for maps of South America and Africa, and sometimes for world maps. It was combined with the Mollweide by Goode (1925) to create the Homolosine projection.

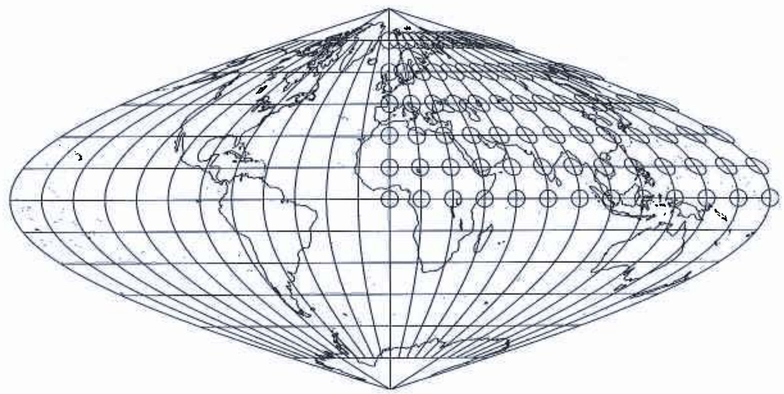


Figure 8-19 The Sinusoidal projection.

8.4.3.4.3 Robinson

Presented by Arthur H. Robinson in 1963 at the request of Rand McNally and Company, the Robinson projection is a pseudocylindrical projection that is neither conformal nor equal area. It uses a set of tabular coordinates rather than mathematical formulas to project coordinates. Robinson created it to improve the world view. The central meridian is a straight line 0.51 the length of the Equator. Other meridians are equally spaced, and resemble elliptical arcs concave toward the central meridian. Parallels are equally spaced straight parallel lines between 38° North and South, with space decreasing beyond these latitudes. The poles are shown as lines 0.53 times the length of the Equator. Scale is true along the 38° latitudes North and South, and is constant along any given latitude. There is no point completely free of distortion, and both size and shape change as shown by the circles in Figure 8-20. The Robinson projection is used for world maps by Rand McNally in their *Goode's World Atlas* (Veregin 2006).. The National Geographic Society adopted it for world maps for a time during the 1990s.

8.4.3.5 Other Projections

Regions that are not predominately elongated along the cardinal directions, not circular in shape, and too large for local projections present a conundrum to the cartographer and the geodesist. The Transverse cylindrical of Professor Rosenmund for Switzerland (Mugnier 2001) and the Oblique Mercator of French General Jean Laborde for Madagascar (Mugnier 2000) were based on double projections that utilized an equivalent sphere. Laborde's development is based on the Gauss-Schreiber Transverse Mercator projection. Hotine (1946, 1947) first introduced the development of the oblique Mercator on the ellipsoid through the "aposphere," a surface of constant curvature and thence to the plane. Hotine's original implementation was for the peninsula of Malaya and the island of Borneo, but this projection also has been used as a grid in numerous areas elsewhere, including Alaska Zone 1 of the State Plane Coordinate Systems on both NAD 27 and NAD 83.

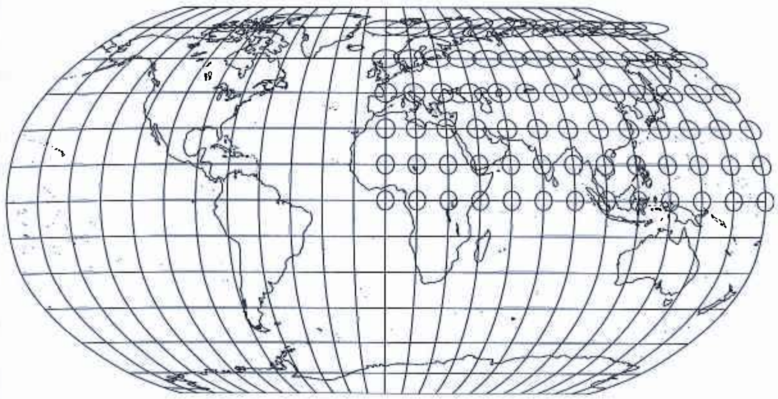


Figure 8-20 The Robinson projection.

8.4.3.5.1 Van der Grinten

The Van der Grinten projection (also called Van der Grinten 1), presented by Alphons J. van der Grinten of Chicago in 1898, is a polyconic projection that is neither conformal nor equal area. The projection has a straight central meridian whereas other meridians are circular and equally spaced along the Equator, concave toward the central meridian. Parallels are circular arcs, concave toward the nearest pole, with the Equator as a straight line exception. The Poles are points. Scale is true along the Equator and increases rapidly with distance from the Equator. The projection has significant distortion near the poles (Figure 8-21). The projection encloses the entire world in a circle. The US Department of Agriculture, the USGS, and the National Geographic Society are a few of the organizations that have used it for world maps.

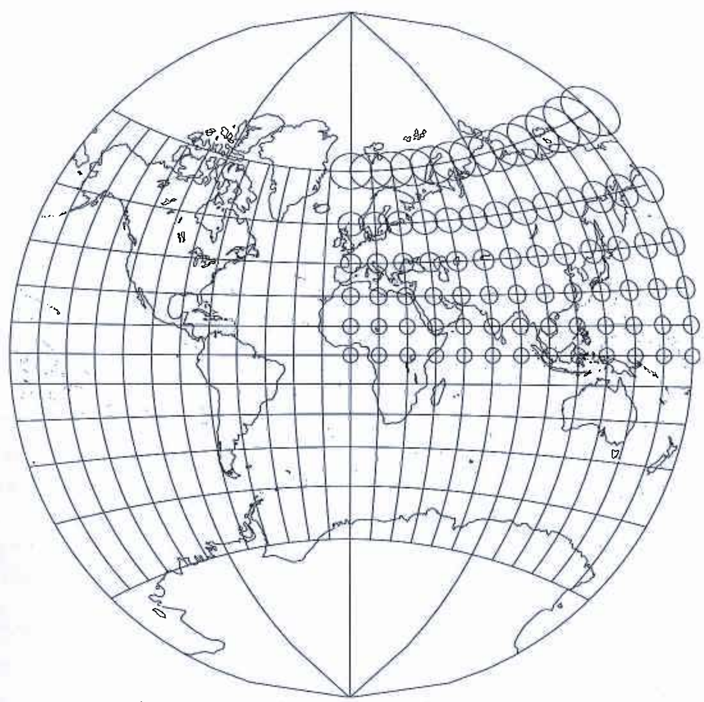


Figure 8-21 The Van der Grinten projection.

8.4.3.5.2 Cassini-Soldner

The Cassini-Soldner projection is a relic of the 19th century mapping efforts of the Europeans and their colonies (Clark 1973; Iliffe 2000). Largely replaced by the Transverse Mercator Projection, the Cassini-Soldner occasionally is still found in former British colonies that describe cadastral records and/or hydrocarbon exploration/production concessions with this grid. The Cassini-Soldner is an aphylactic projection also in that it is neither conformal nor equivalent. Survey computations on the developed surface are especially problematic, particularly with respect to the conversion between geodetic distances measured on the ground, and grid distances measured on the developed surface. The Cassini-Soldner projection centered over the zero degree latitude and longitude is shown in Figure 8-22.

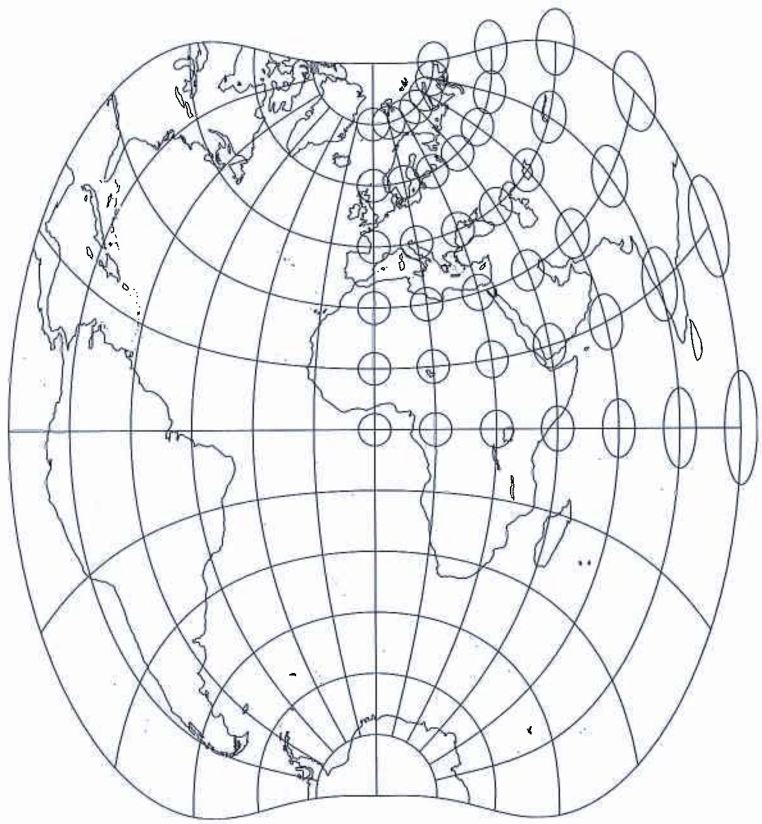


Figure 8-22 The Cassini-Soldner projection centered on zero degrees latitude and longitude. Note that only a part of the Earth appears in the projection.

8.4.3.5.3 Space Oblique Mercator

The Space Oblique Mercator projection, conceived by Alden P. Colvocoresses in 1973 and developed mathematically by John P. Snyder in 1977, was designed to map the ground track of a satellite and maintain conformality. Meridians and parallels are complex curves at slightly varying intervals to account for the motion in time of the satellite (Figure 8-23). The Poles are points. The scale is true along the ground track, but varies about 0.01 percent within the normal sensing range of the satellite. There is no distortion along the ground track and distortion is constant along lines of constant distance parallel to the ground track. The projection is conformal to within a few parts per million for the sensing range. The projection is used for satellite images including Landsat and others.

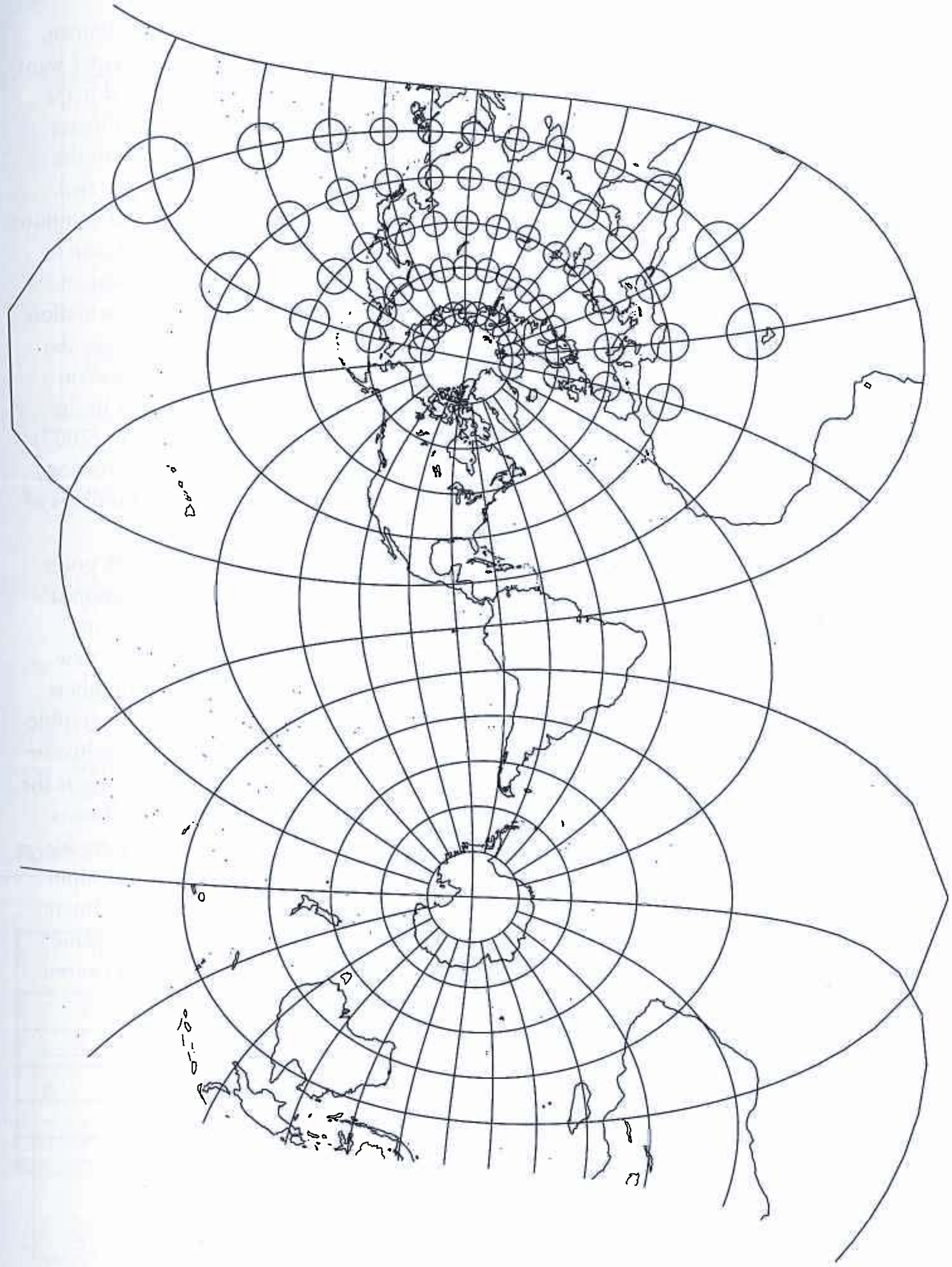


Figure 8-23 The Space Oblique Mercator protection conceived by Alden Colvocoresses and derived mathematically by John P. Snyder.

8.5 Plane Coordinate Transformation

As GIS users are well aware, geographic data often exist in a plane coordinate system, but not in the particular plane system we wish to use. If the datum, projection, and associated parameters are known, the data can be reprojected to the desired projection and coordinate system. In this situation, the data are first inversely projected to the geographic reference system of latitude and longitude, and then forward projected to the desired system. This reprojection operation allows exact control of the transformation, and the accuracy and errors involved.

The data are frequently in plane coordinates, but the datum or projection is not known. An example is a photograph or image that exists in an image coordinate system that we want to transform to UTM to match other data. Other examples are digitized or scanned maps and images. For these data, an approximate transformation to a set of known coordinates can be performed. A common approach is to locate ground control points (GCPs) in the image and a reference system and develop a polynomial transformation between the two. Coordinates in the image system can be located visually and measured directly on a computer screen. Coordinates of the same points in a reference system can be located from a map or in the field with a GPS receiver. The mapping of points between the two systems and an explanation of the process for establishing the equations to be solved for the transformation is illustrated in Figure 8-24 and 8-25, respectively. The mapping in Figure 8-25 shows the process to transform an image scanned at 1,024 by 1,024 pixels to UTM coordinates on NAD 83. The root-mean-square error of the transformation is ±0.69 m. Note that in the example a simple first order polynomial is used, which is sufficient for most current (2007) geographic data sets. Higher order polynomials can be used to eliminate higher distortion, but require larger numbers of control points. The minimum and recommended numbers of GCPs for polynomials of degrees 1-5 are shown in Table 8-2.

The described transformation between plane coordinate systems can be used with point (vector) data or image (raster) data representations. For vector data, the transformation is complete since attributes are associated with the transformed points, lines, or polygons. For raster data, since we are transforming discrete cells, we must determine how the new cell values will be assigned. For these data, we must resample the original digital numbers (DNs) or raster cell values to match the new geometry of the transformed image. A graphic example of the concept of this resampling is shown in Figure 8-26. It is shown as an inverse resampling, from the output coordinate space to the input coordinate space, since this is the common implementation. The exact raster organization desired, i.e., the number of rows, the number of columns, and the pixel size is assumed, and the image is mapped from this assumed space back to the original coordinate space. Once the exact location in the input space is known, the appropriate DN or raster cell value is placed in the output coordinate (pixel) position. Common resampling approaches are nearest neighbor (assume the value of the closest pixel), bilinear interpolation (a distance-weighted average of the four nearest values), or cubic convolution (a distance-weighted average of the 16 nearest pixels). Details of these resampling methods are available in a variety of sources (Jensen 2005). Steinwand et al. (2005) have developed resampling techniques for thematic (categorical) data that allow users to select minimum, maximum, modal and other statistical or user-specified values from those available for the sample area in the input raster dataset. The techniques provide significantly better output results than traditional nearest neighbor methods.

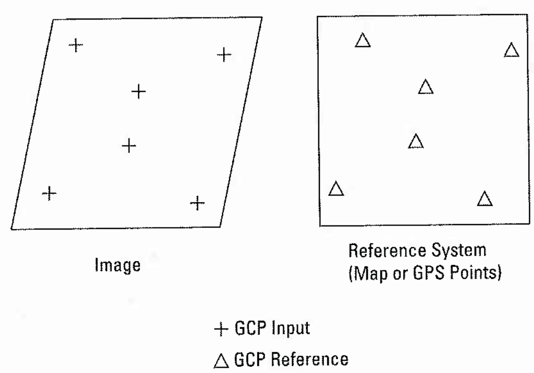
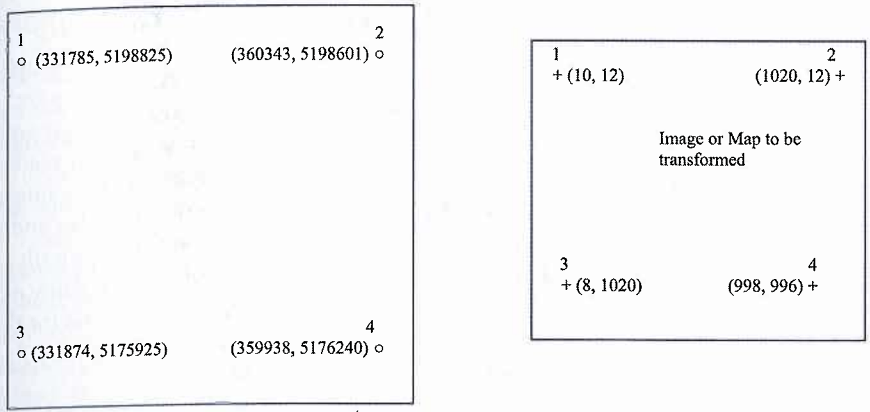


Figure 8-24 Mapping of points between two raster GIS or image and map systems using a series of ground control points (GCPs) (Welch and Usery 1984).



$$X^t = a_0 + a_1 x + a_2 y$$
$$Y^t = b_0 + b_1 x + b_2 y$$

From the first point:

$$331785 = a_0 + a_1(10) + a_2(12)$$
$$5198825 = b_0 + b_1(10) + b_2(12)$$

From the second point:

$$360343 = a_0 + a_1(1020) + a_2(12)$$
$$5198601 = b_0 + b_1(1020) + b_2(12)$$

From the third point:

$$331874 = a_0 + a_1(8) + a_2(1020)$$
$$5175925 = b_0 + b_1(8) + b_2(1020)$$

From the fourth point:

$$359938 = a_0 + a_1(998) + a_2(996)$$
$$5176240 = b_0 + b_1(998) + b_2(996)$$

Solve for the unknowns a_0, a_1, a_2, b_0, b_1, b_2 by simultaneous solution with least squares adjustment. Apply coefficients to all other points in the input image to create complete transformed geometry of the image. For raster images, resample to generate new gray level or color values.

Figure 8-25 Implementation method for polynomial transformation for image or map data.

Table 8-2 Number of GCPs for plane coordinate transformation

Polynomial Order	Minimum Number of GCPs	Recommended Number for Effective Least Squares
1	3	6
2	6	10
3	10	15
4	15	21
5	21	30

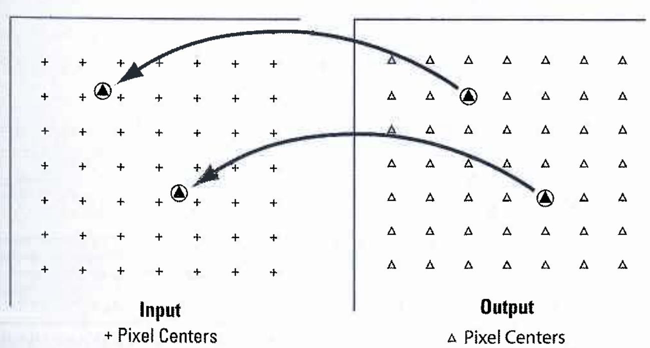


Figure 8-26 Resampling example using nearest neighbor concept from the output image mapped into the input image.

8.4.1 Three-Dimensional to Two-Dimensional Transformations

Three-dimensional (3D) and two-dimensional (2D) transformations occur with geographic data since the surface of the Earth is not a perfect sphere or ellipsoid. Thus, data acquired over a 3D surface must be transformed to a plane representation. The transformation process is shown in Figure 8-27. As with map projection, this transformation is completely mathematical and model error sources can be determined exactly. Figure 8-28 provides an image example. A photograph of the Tenth Legion, Virginia, area is shown in Figure 8-28a and the same photograph after it has been orthorectified to remove distortion resulting from tilt and relief is shown in Figure 8-28b. Note that roads on the photo appeared curved because of terrain relief, but in the orthophotograph the roads appear straight. The procedures for these transformations are the subject of photogrammetry and are explained in complete detail in the ASPRS *Manual of Photogrammetry* (McGlone 2004).

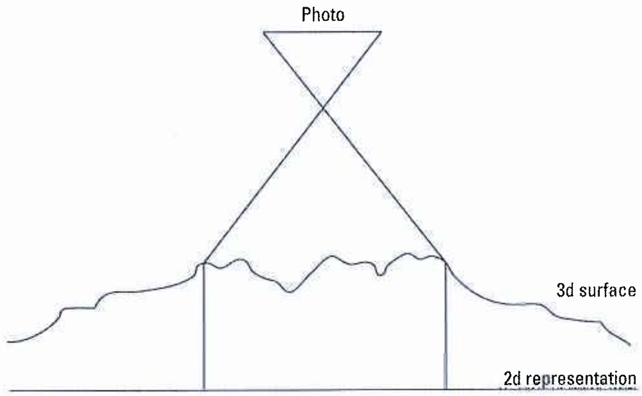


Figure 8-27 Graphic example of the 3D to 2D transformation process.

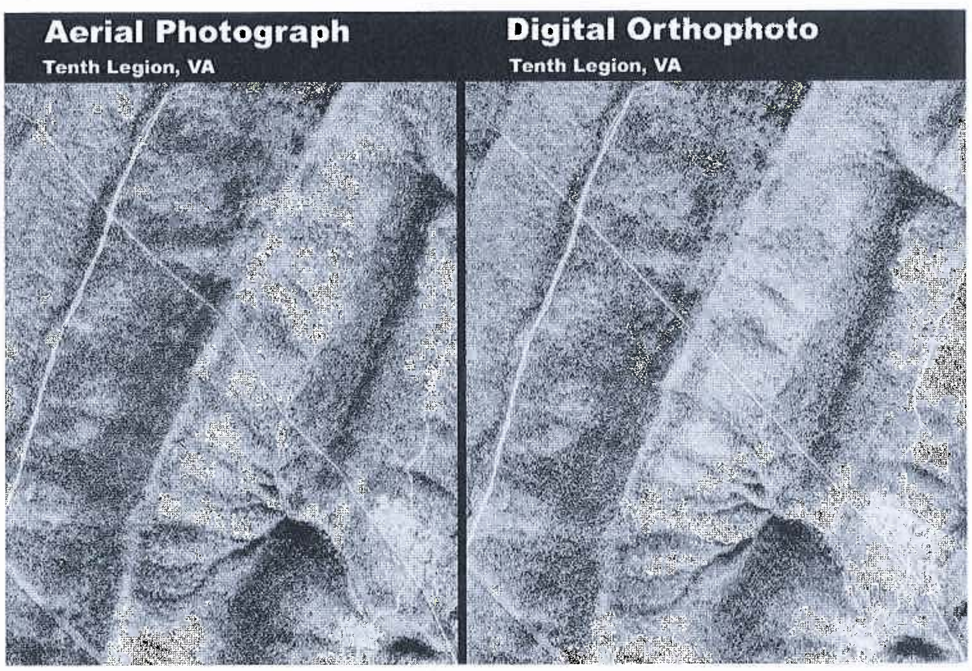


Figure 8-28 Photo example of the 3D to 2D transformation. Note the road appears curved in the (left) uncorrected image and straight in the (right) terrain-corrected image.

8.6 Conclusions

Coordinate transformations are the basis of achieving a common frame of reference for geographic information analysis in GIS. The requirement of a common ellipsoid, datum, map projection, and finally plane coordinate system make it possible to use plane geometry for all types of spatial overlay and analysis. The methods of transformation are many and varied, and can be accomplished as rigorous mathematical transformations or as simple approximations. The accuracy of the resulting analysis, however, can only be as good as the accuracy of the data. Geographic data projection from the ellipsoidal Earth to a plane coordinate system always results in error in area, shape, and other properties. With appropriate selection of a projection, the user can preserve desired characteristics at the expense of others. In this chapter, basic concepts of coordinate systems and map projections were examined. For a more in-depth treatment, the reader is referred to the texts and sources referenced in this section and listed below.

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